Beyond SAT and SMT
Automated Reasoning Building Blocks

Learn Fresh

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Reasoning Systems

Key Isabelle Coq PVS KeYmaera VCC

Automated Reasoning Systems

beagle SPASS(T)

princess iSAT

SPASS

Z3 CVC4 Barcelogic

E Vampire

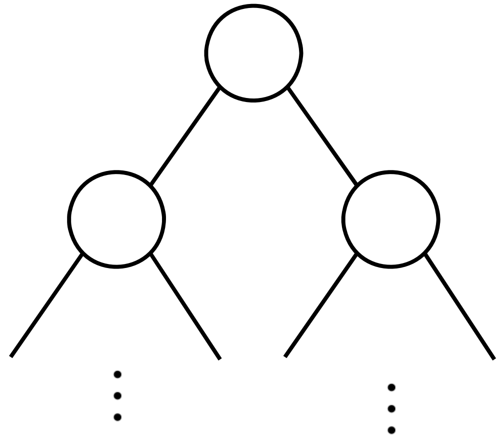
MathSat Yices

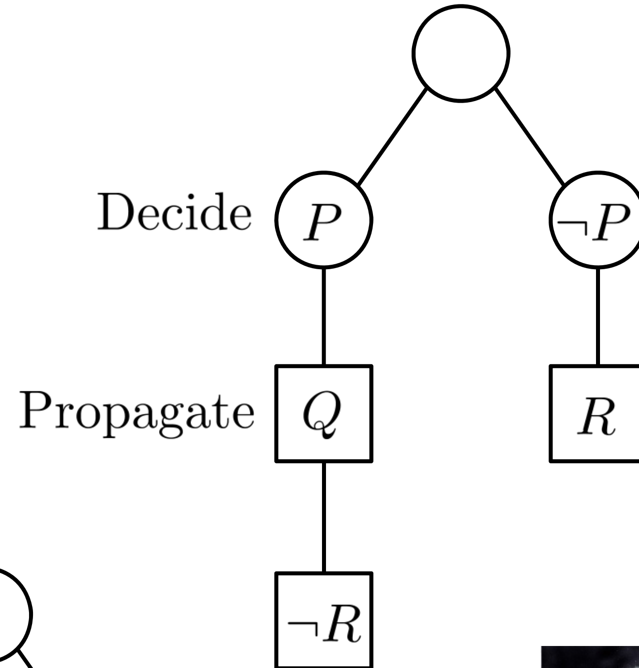
MiniSat Lingeling

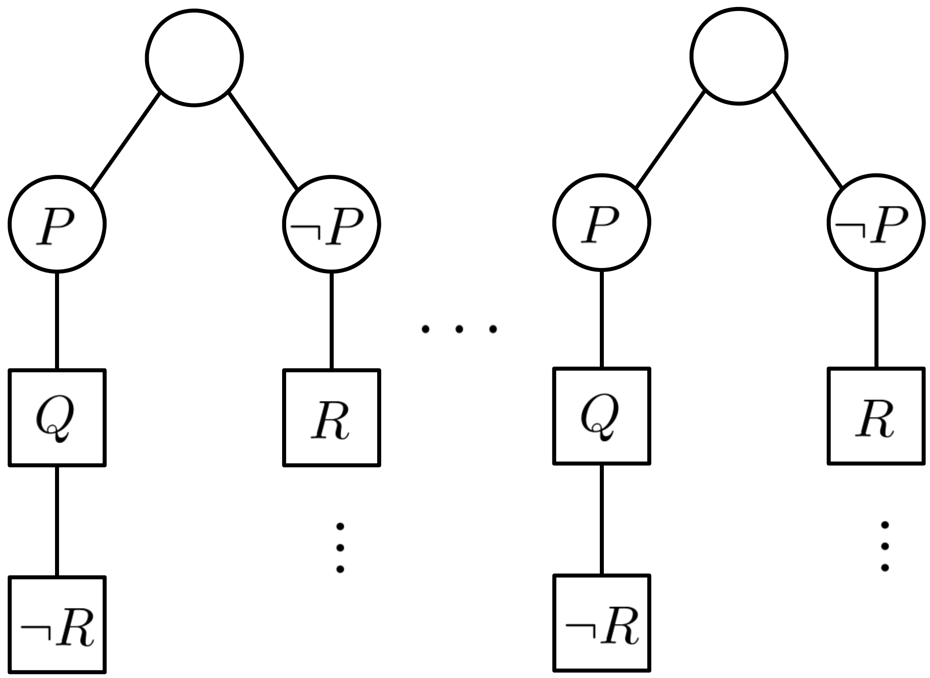
Sat4j zChaff

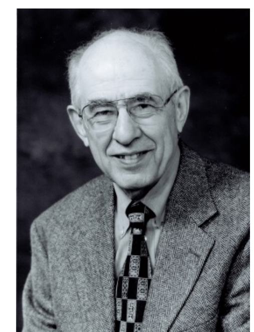
DPLL 1962

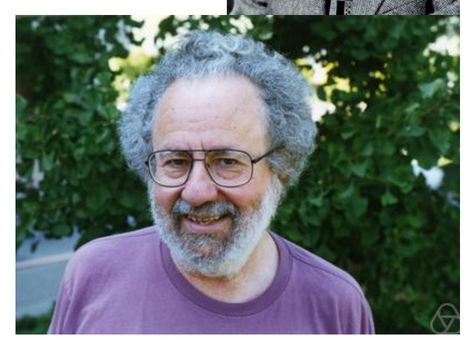
$\neg P \lor \neg Q \lor R$
$\neg P \lor Q$
$\neg P \lor \neg R$
$P \lor R$











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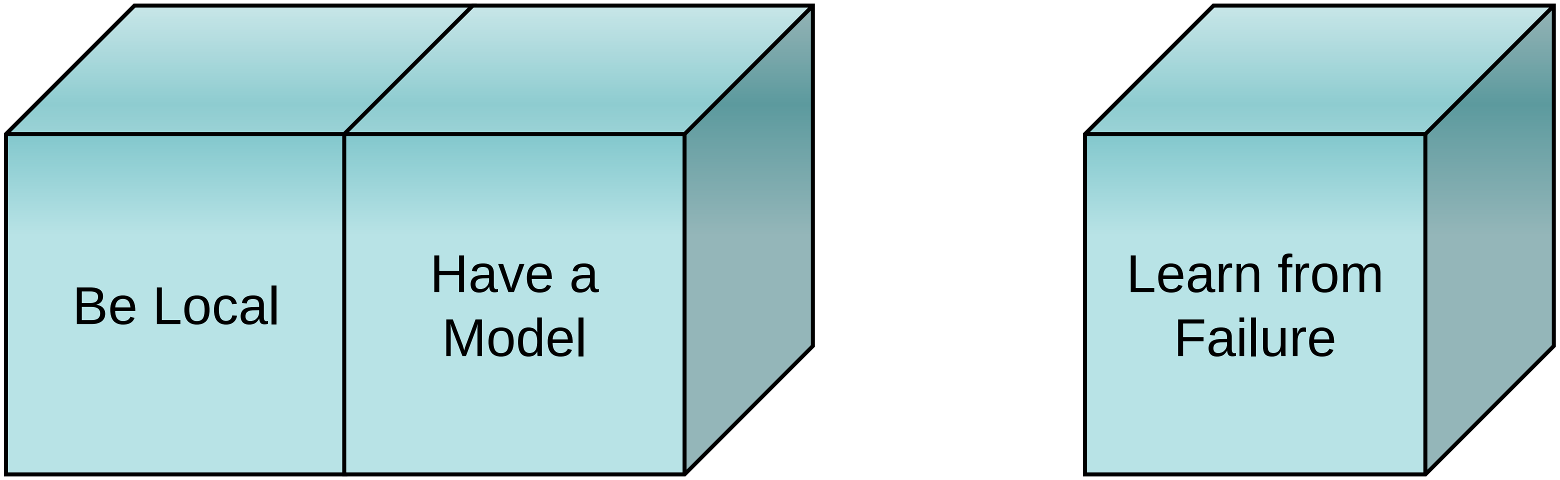


Bachmair Ganzinger Superposition 1990

$\neg P \lor \neg Q \lor R$ total ordering on literals: $R \prec \neg R \prec P \prec \neg P \prec Q \prec \neg Q$

$\neg P \lor Q$ model assumption: $\neg R$, $P^{P \lor R}$, $Q^{\neg P \lor Q}$

$\neg P \lor \neg R$

$P \lor R$

$$\frac{\neg P \lor \neg Q \lor R \qquad \neg P \lor Q}{\neg P \lor R}$$

model assumption: $\neg R$, $P^{P \lor R}$

$$\frac{\neg P \lor R \qquad P \lor R}{R}$$

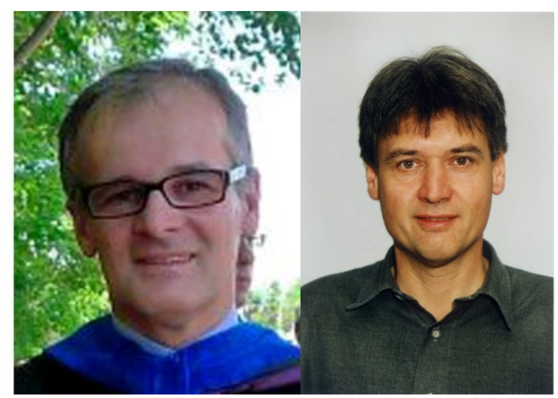
model assumption: $R, \ldots$

$\neg P \lor \neg Q \lor R$ $\neg P \lor R$

$\neg P \lor Q$ R

$\neg P \lor \neg R$

$P \lor R$



max planck institut informatik

Bachmair Ganzinger 1990 Continued

$\neg P \vee \neg Q \vee R$
$\neg P \vee Q$
$\neg P \vee \neg R$
$P \vee R$
$\neg P \vee R$
R

total ordering on literals: $R \prec \neg R \prec P \prec \neg P \prec Q \prec \neg Q$

model assumption: $R, \ldots$

C redundant if $D_1, \ldots, D_n \models C$ and $D_i \prec C$

$\neg P \vee R$ because implied by R and $R \prec \neg P \vee R$

$P \vee R$ because implied by R and $R \prec P \vee R$

$\neg P \vee \neg Q \vee R$ because implied by R and $R \prec \neg P \vee \neg Q \vee R$

$\neg P \vee Q$
$\neg P \vee \neg R$
R

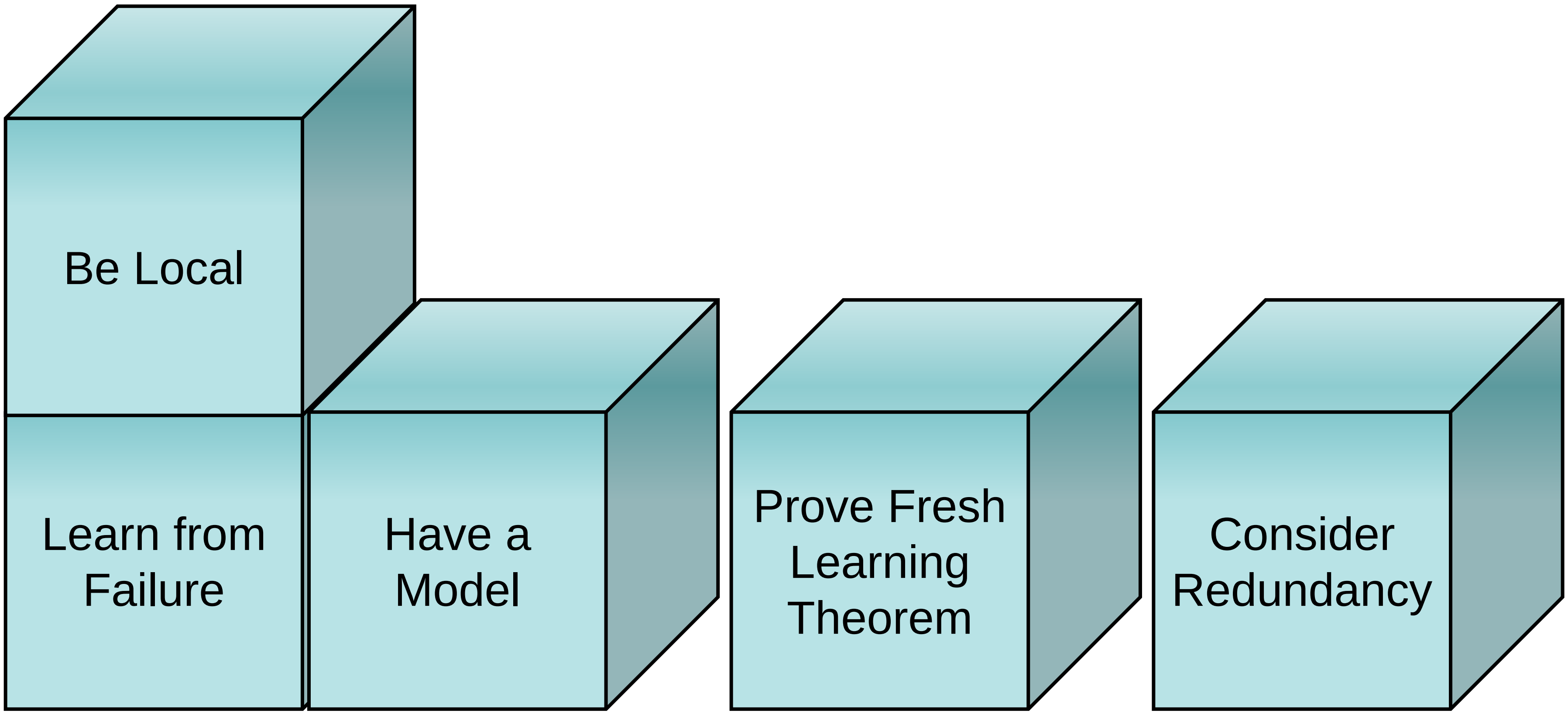
$\neg P \vee Q$ because implied by $R, \neg P \vee \neg R$ and $R, \neg P \vee \neg R \prec \neg P \vee Q$

$\neg P \vee \neg R$
R

Fresh Learning Theorem

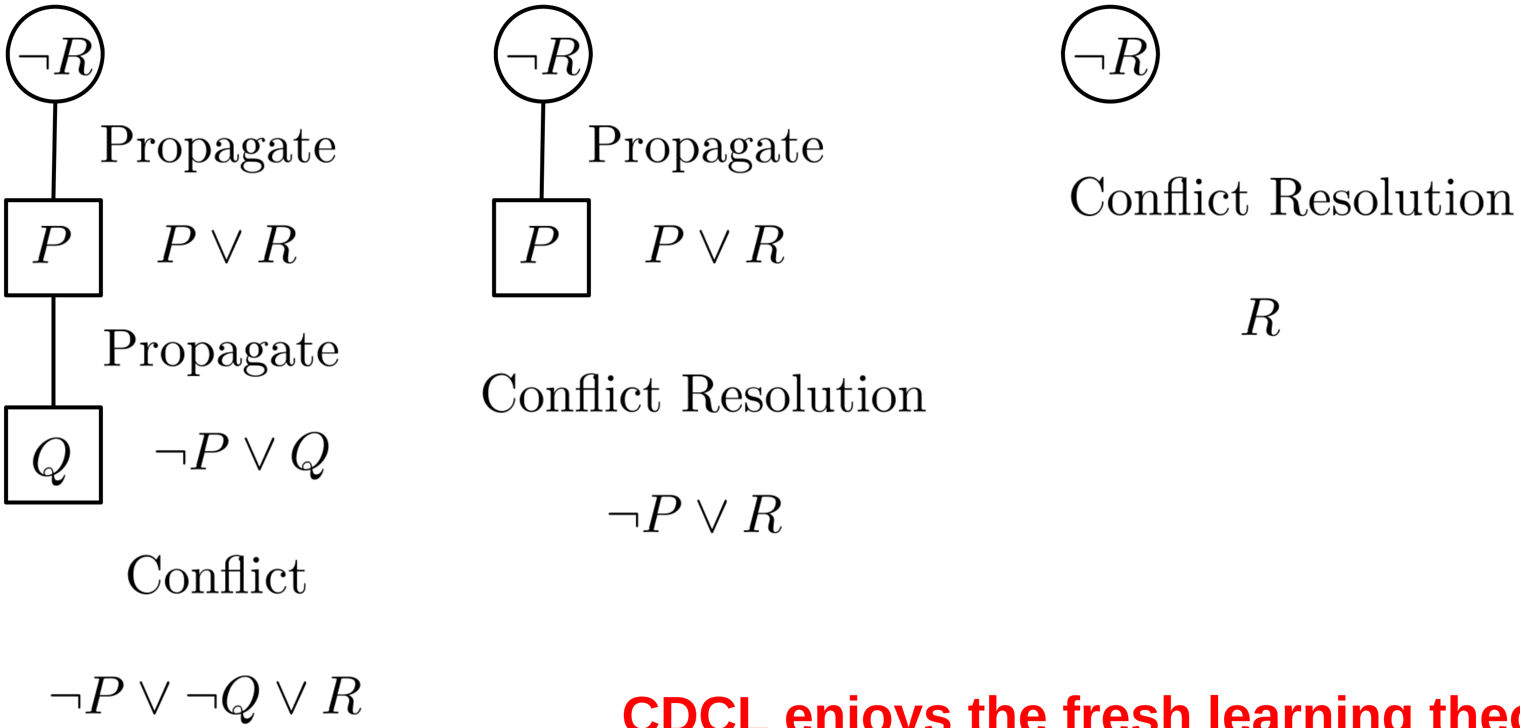
**The clauses
learned out of a failed model assumption
are not redundant.**

Automated Reasoning Building Blocks



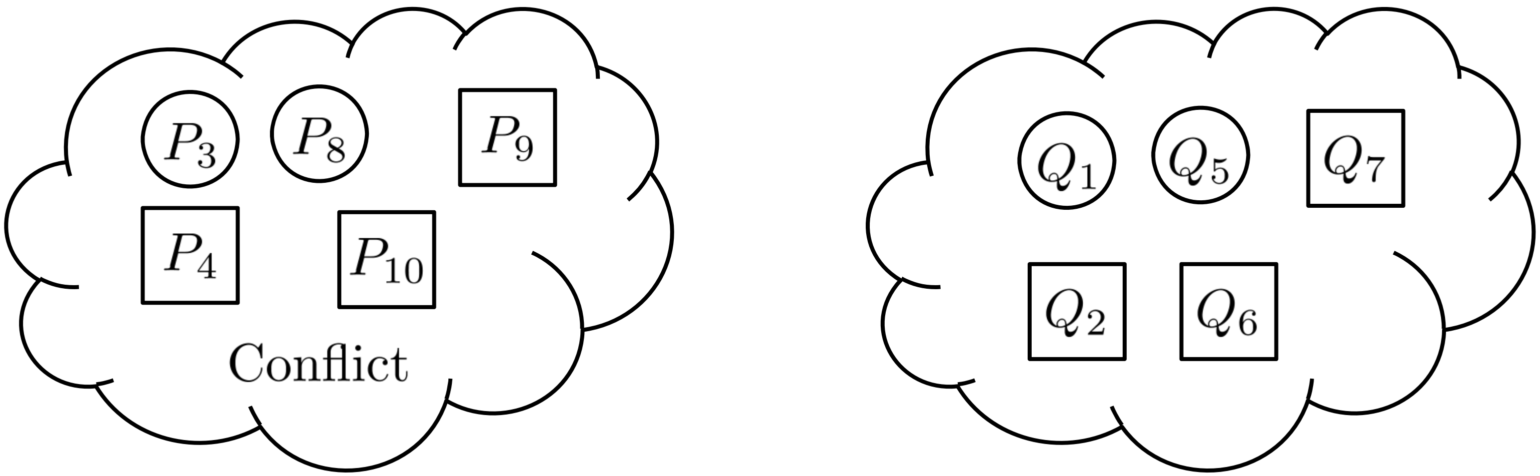
CDCL 2000-today

$\neg P \lor \neg Q \lor R$
$\neg P \lor Q$
$\neg P \lor \neg R$
$P \lor R$



CDCL enjoys the fresh learning theorem.

CDCL Ordering Change



ordering so far $Q_1 \prec Q_2 \prec P_3 \prec P_4 \prec Q_5 \prec \ldots$

after conflict resolution $P_i \prec Q_j$ for all P_i involved in the conflict

bonus for all literals involved in the conflict, penalty for the others

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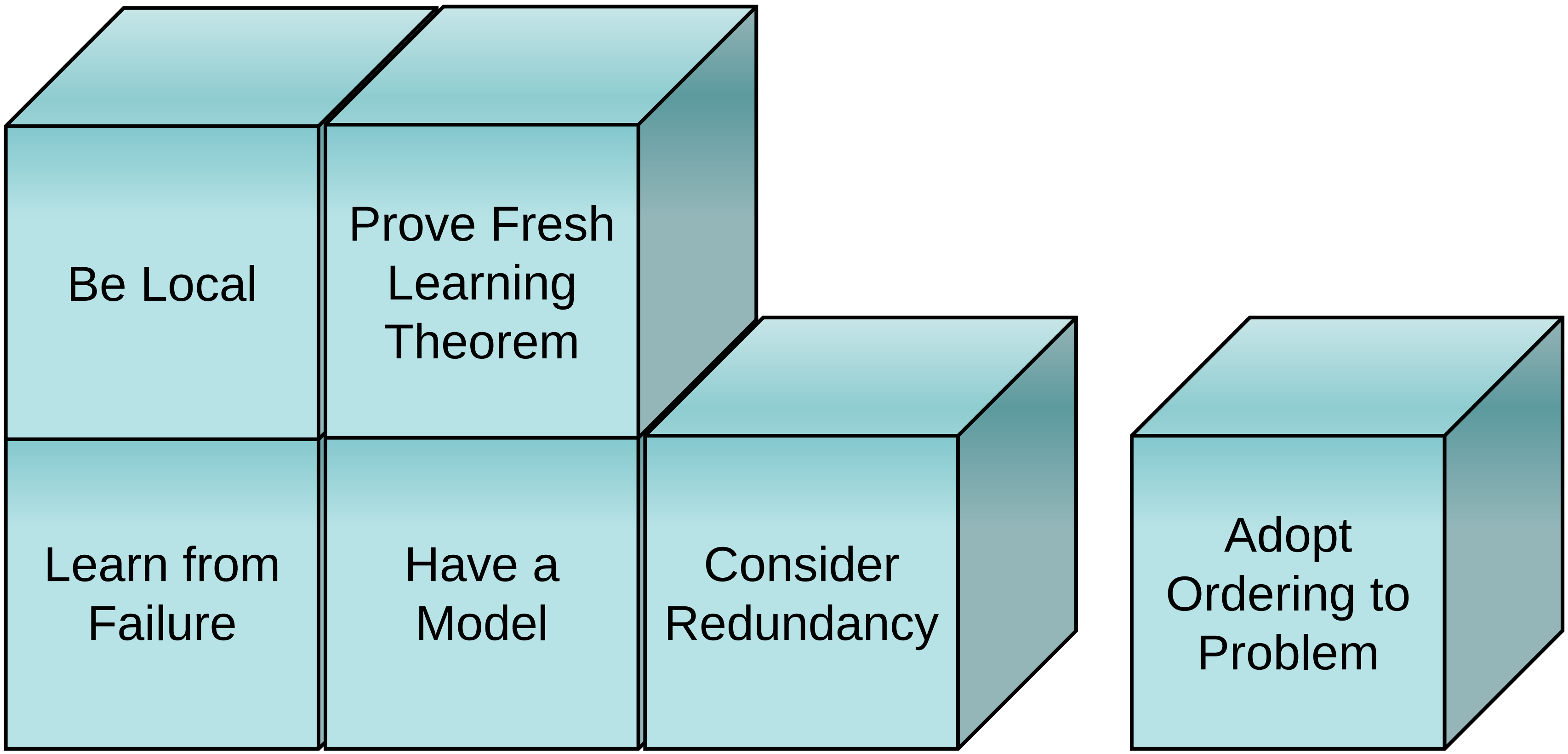


Dynamically Changing the Ordering

- finitely often, no problem
- for otherwise no completeness, termination

$$\frac{P \vee \neg Q \qquad Q \vee R}{P \vee R}\; R \prec P \prec Q$$

but $P \vee R$ is redundant with ordering $Q \prec P \prec R$

But why does this work for CDCL?

- use redundancy notion invariant to ordering changes (length)
- provide a different ordering for completeness (total number of clauses)

Computational Aspects

Model Representation M

Sequence of literals $M = \neg R,\ P,\ Q$

Propagation: for some clause $[\neg]P \vee C$ decide $M \not\models C$, P undefined

$M = \neg R,\ P$ clause $\neg P \vee Q$ propagates Q

Conflict: for some clause C decide $M \not\models C$

$M = \neg R,\ P,\ Q$ clause $\neg P \vee \neg Q \vee R$ is false

Conflict Resolution: compute consequences out of false clause

Redundancy: decide $D_1, \ldots, D_n \models C$

For first-order logic not effective in general.

Jovanovic, de Moura 2012: Polynomials

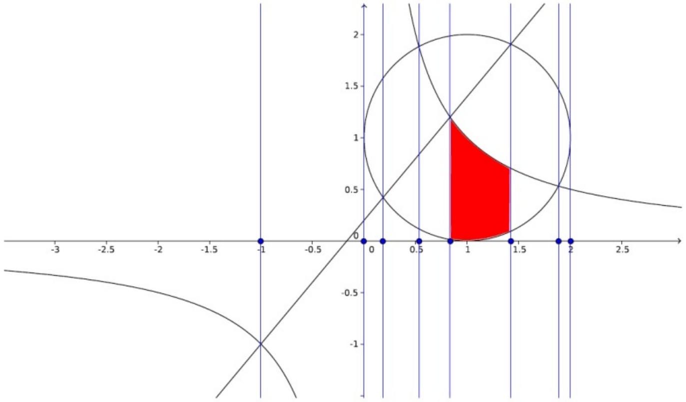
Set of polynomials $3\,x_1^3\,x_2 + 5\,x_3^6\,x_1\,x_2 \leq 0$ find a solution.

Model Representation M: assignment of values to $x_1, \ldots, x_k$

Propagation: $M = a_1, \ldots, a_k$ polynomial in $x_1, \ldots, x_{k+1}$ compute a_{k+1}

Conflict: $M = a_1, \ldots, a_k$ violate some disequation

Conflict Resolution: use CAD to learn the conflict cell



The calculus enjoys the fresh learning theorem.

Finite Domain FOL(T)

Is a first-order clauses set over LA and some finite domain a_1, a_2 satisfiable?

Clause: $P(x, y) \lor 3x + 2y > 0$

Clauses:
$$\begin{array}{l} P(a_1, a_1) \lor 3a_1 + 2a_1 > 0 \\ P(a_1, a_2) \lor 3a_1 + 2a_2 > 0 \\ P(a_2, a_1) \lor 3a_2 + 2a_1 > 0 \\ P(a_2, a_2) \lor 3a_2 + 2a_2 > 0 \end{array}$$

Grows exponentially in number of different variables

For three different variables and n elements n^3 may get too large.

Reasoning with $P(x, y) \lor 3x + 2y > 0$ can be exponentially better.

Alagi, Weidenbach: Finite Domain FOL(T) 2012-

First-order clause set over some finite domain $a_1, \ldots, a_k$, satisfiable?

Model Representation M: sequence of constrained literals $(P(x,y), x \neq y))$

Propagation: for some clause $[\neg]A \vee C$ decide $M \not\models C\sigma$, $A\sigma$ undefined

Conflict: for some clause C decide $M \not\models C\sigma$

Conflict Resolution: compute consequences out of false clause

Redundancy: decide $D_1, \ldots, D_n \models C\sigma$

The calculus enjoys the fresh learning theorem.

Working on practically efficient algorithms.

Automated Reasoning Building Blocks

Thanks for your attention!