



# Energy as Syntax

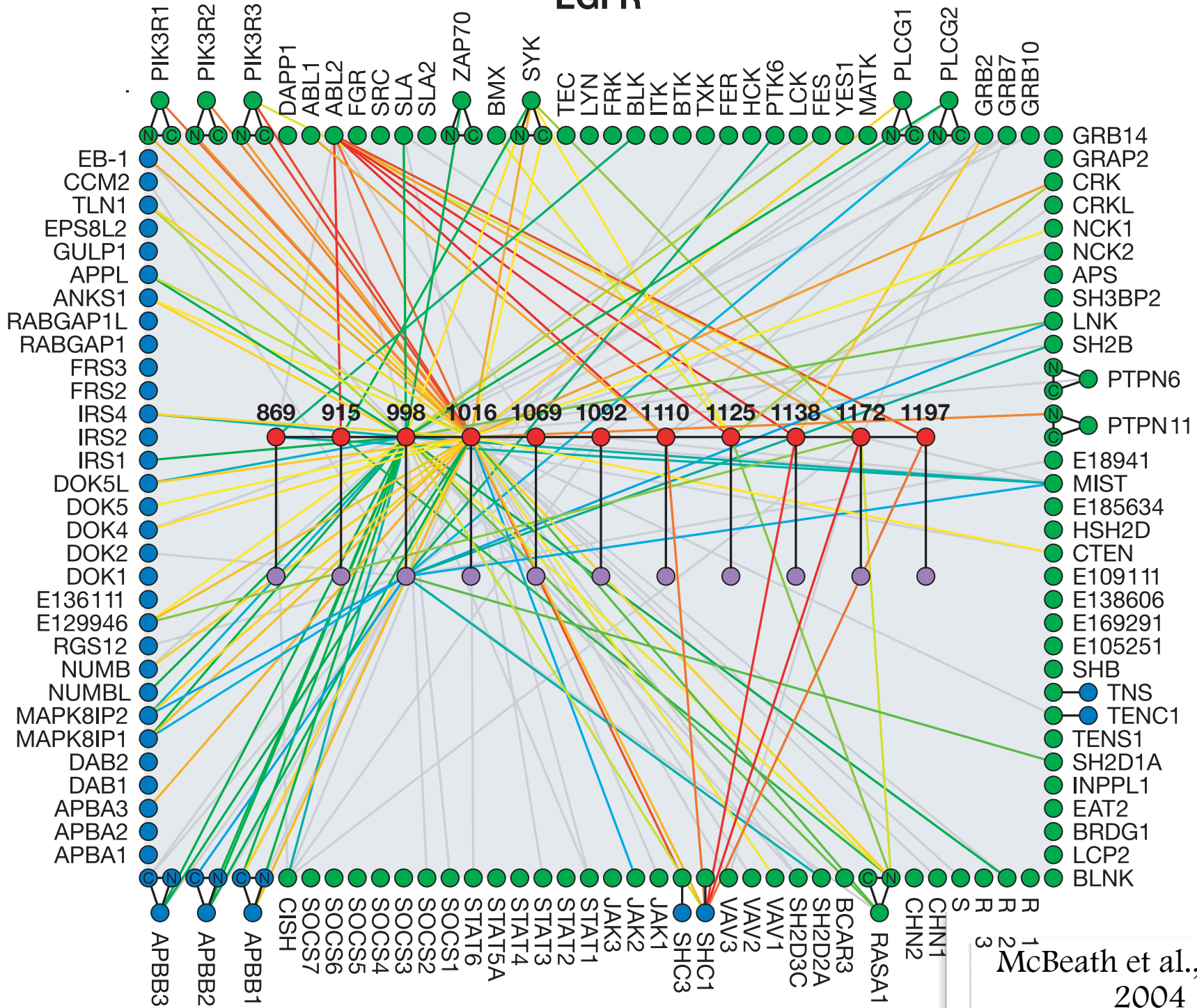
2-19 CMACS lecture

joint work with J. Ollivier, N. Oury, E. Lai

# turbo introduction to Kappa/BNGL

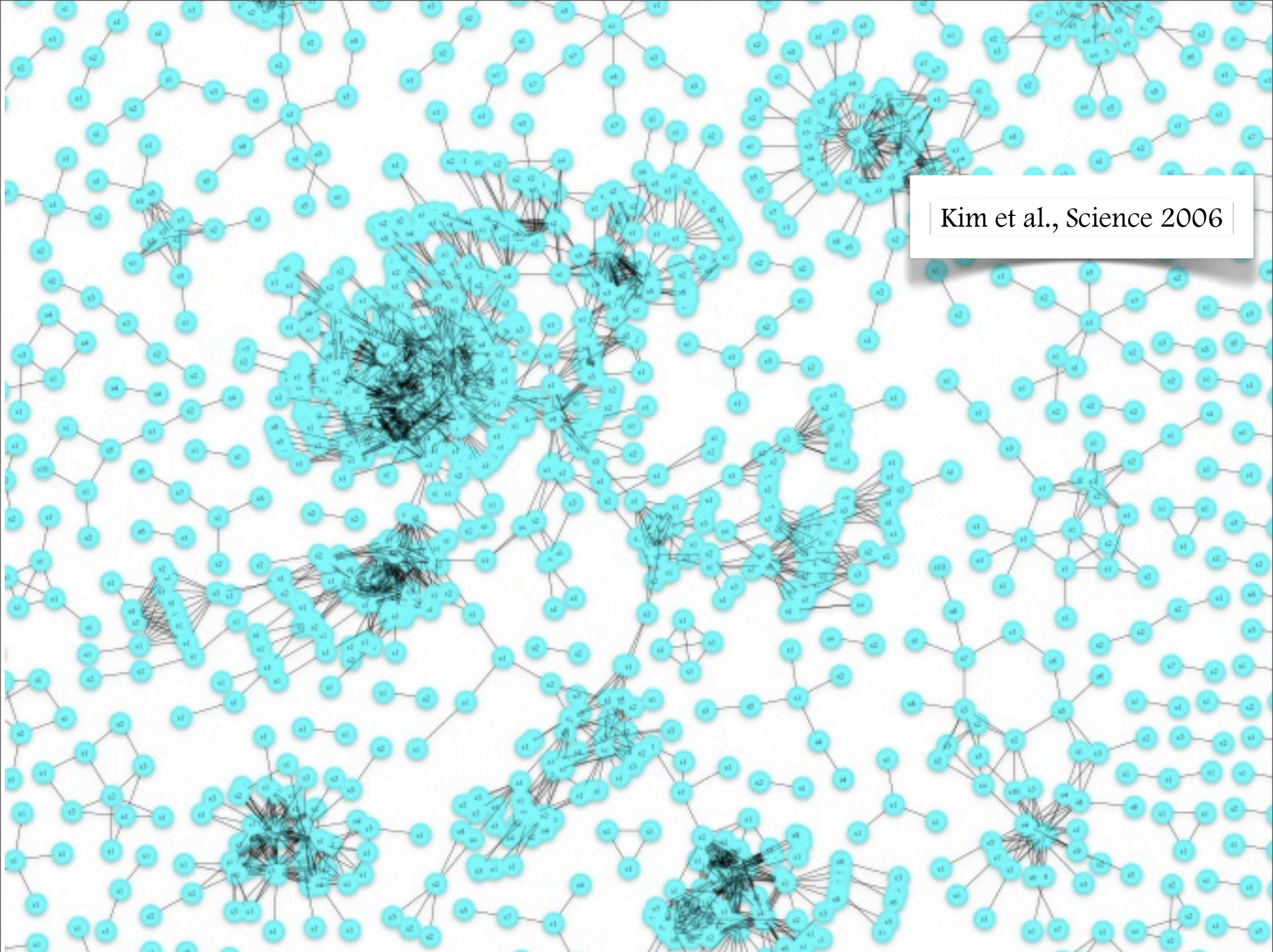
why rules are great

# EGFR



McBeath et al., Nature  
2004



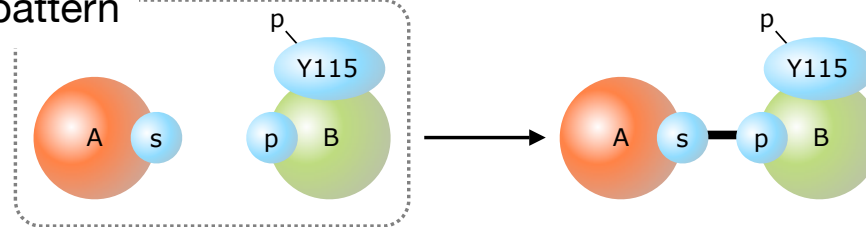


Kim et al., Science 2006

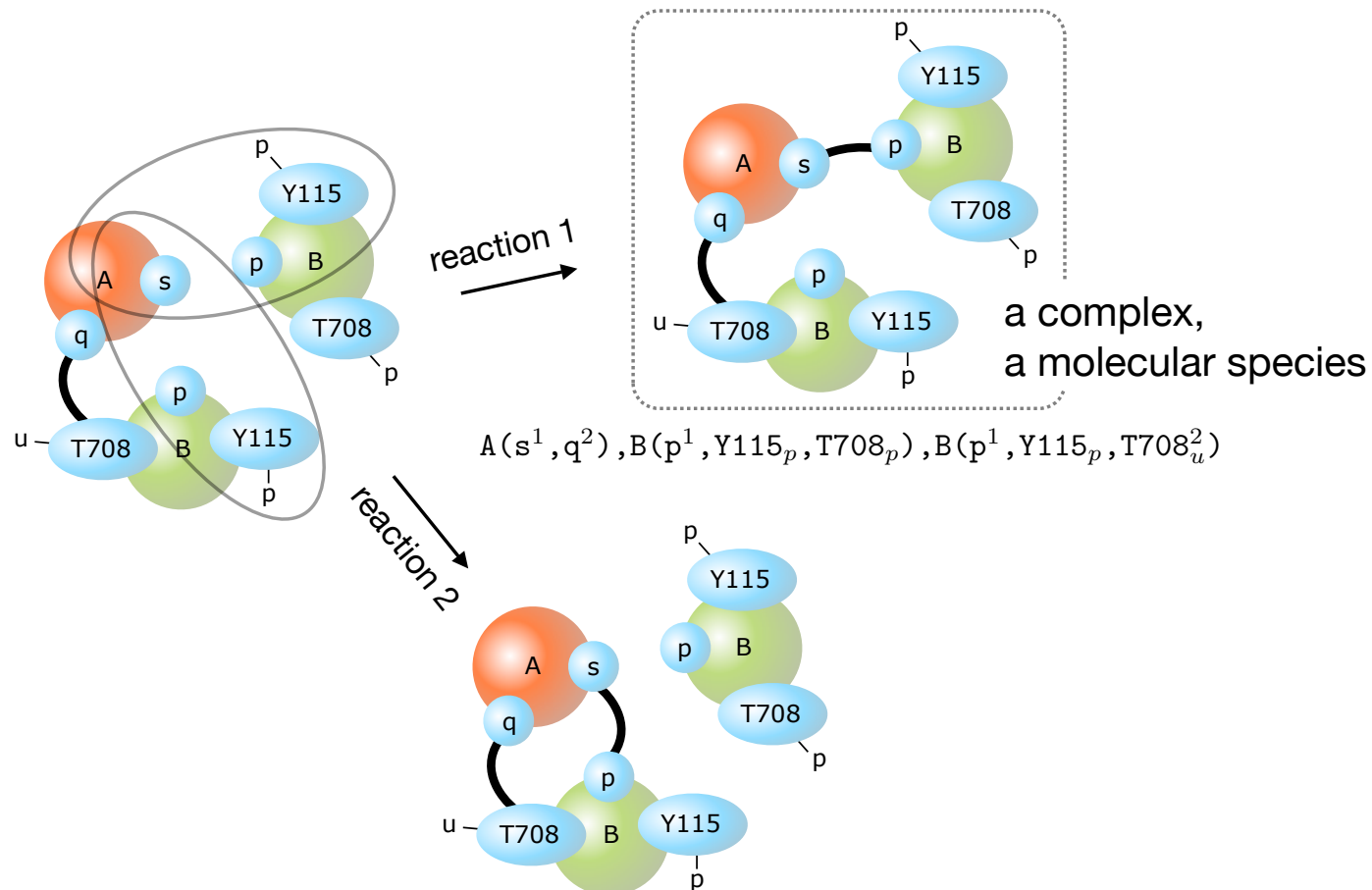
# probabilistic (site) graph rewriting

a rule

a pattern



$$A(s), B(p, Y115_p) \rightarrow A(s^1), B(p^1, Y115_p)$$



# try Kappa!

[http://www.pps.jussieu.fr/~jkrivine/  
binaires/Telechargements/  
Outils\\_danalyses\\_et\\_de\\_simulation.html](http://www.pps.jussieu.fr/~jkrivine/binaires/Telechargements/Outils_danalyses_et_de_simulation.html)

forget about rules!

# energy as syntax

energy-oriented modelling/programming

– more structured approach

- as in structured programming
- esp. well suited for combinatorial molecular network for which:
- no structure means no analysis possible ...

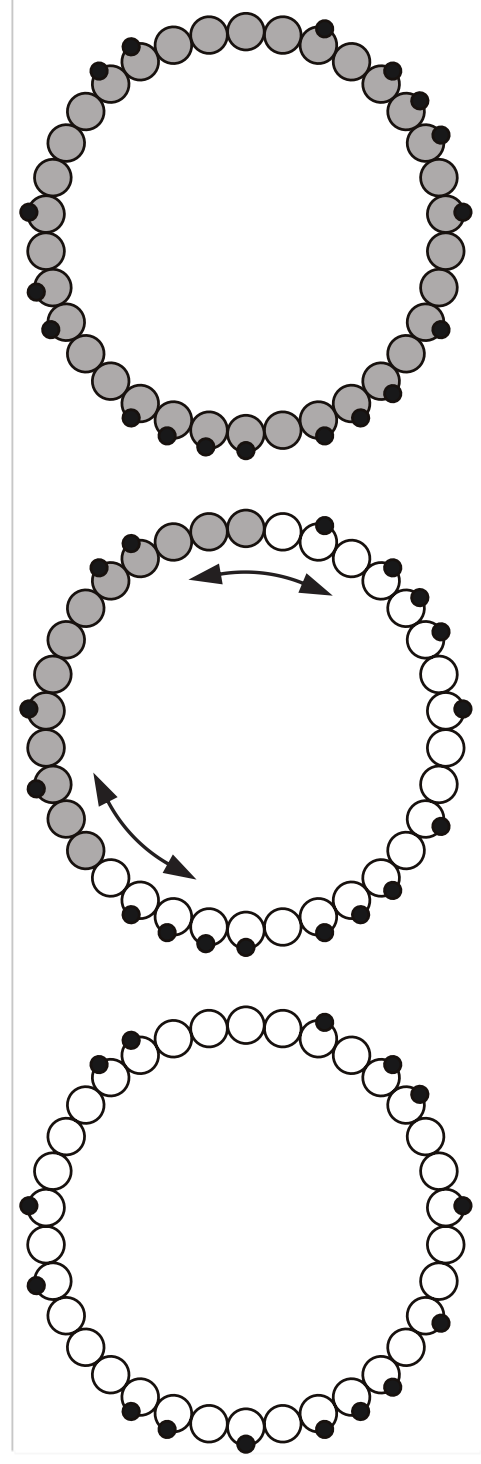
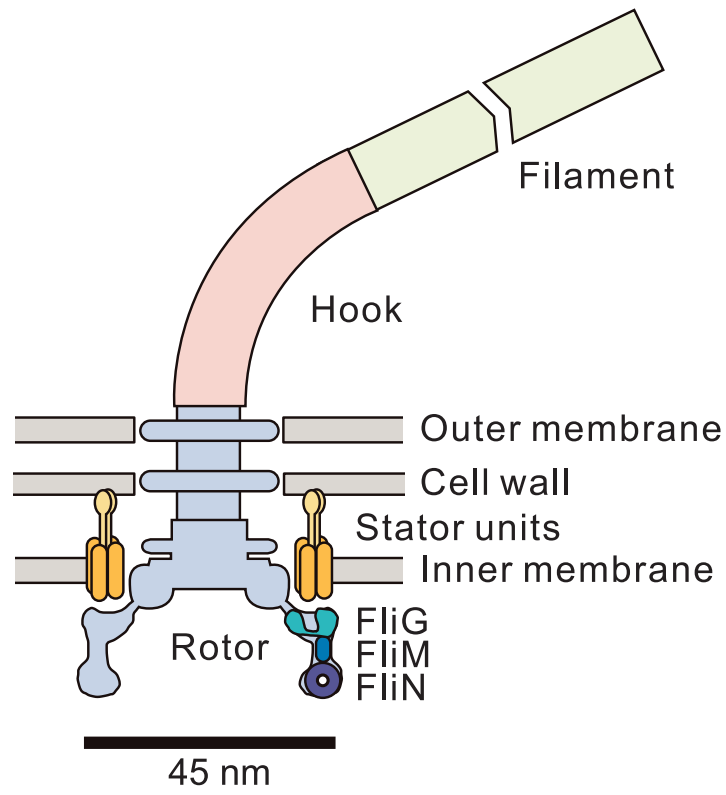
– more physically realistic

– less parameter-hungry



# example: an allosteric Ising model

[Science – Feb 5 2010]



# an allosteric Ising model

an allosteric model of the E. Coli flagellar switch (with ANC-style energy)

- a ring of 2-state protomers  $P(f)$ :
  - ↻ [favoured]  $f=0$ =inactive (counter clockwise)
  - ↻ [disfavoured]  $f=1$ =active (clockwise)
- potential binders CheY that favour  $f=1$

# combinatorics & nn

CheY(s~p) might bind any P, which means an astonishing  $\sim 10^{20}$  different configurations (that is the number of species one would need in a species-centric approach)

we are going to write the Hamiltonian/  
energy of the system – a sum of 3  
different contributions  
all terms are nn=nearest neighbour

# Energy landscaping – i

a P conformational term whereby it is said that P prefers conformation 0

$$E(P(f\sim 0)) < E(P(f\sim 1))$$

convention: lower energy = more favoured



# Energy landscaping – ii

a CheY–P binding term whereby we say that if bound to phosphorylated CheY, P prefers conformation 1

$$E(P(f \sim 0, s \sim 1), \text{CheY}(s \sim p \sim 1)) > E(P(f \sim 0, s \sim 1), \text{CheY}(s \sim p \sim 0))$$

nb: this term overlaps with the first one  $E(P((f \sim 0/1)))$

# Energy landscaping – iii

an Ising penalty term for n.-neighbours not being in the same conformation which will "spread conformation"

$$E(P(f \sim 1, x \neq 1), P(y \neq 1, f \sim 0)) =$$

$$E(P(f \sim 0, x \neq 1), P(y \neq 1, f \sim 1)) >$$

$$E(P(f \sim 0, x \neq 1), P(y \neq 1, f \sim 0)) =$$

$$E(P(f \sim 1, x \neq 1), P(y \neq 1, f \sim 1))$$

# dynamics ii – rules

# 10 reversible rules

## 2 binds

### P–CheY binding: CheY needs to be pho'ed & prefers conformation P(f~1) by a factor of 10

'bind 0'  $P(f\sim 0,s), CheY(s\sim p) \leftrightarrow P(f\sim 0,s!1), CheY(s\sim p!1)@1,10$

'bind 1'  $P(f\sim 1,s), CheY(s\sim p) \leftrightarrow P(f\sim 1,s!1), CheY(s\sim p!1)@1,1$

## 8 flips (aka conformational change)

### 4 P flips without CheY – note that P(f~0) is favoured 2/1

'flip 000'  $P(f\sim 0,y!1),P(x!1,f\sim 0,y!2,s),P(x!2,f\sim 0) \leftrightarrow P(f\sim 0,y!1),P(x!1,f\sim 1,y!2,s),P(x!2,f\sim 0)@1,200$

'flip 100'  $P(f\sim 1,y!1),P(x!1,f\sim 0,y!2,s),P(x!2,f\sim 0) \leftrightarrow P(f\sim 1,y!1),P(x!1,f\sim 1,y!2,s),P(x!2,f\sim 0)@1,2$

'flip 001'  $P(f\sim 0,y!1),P(x!1,f\sim 0,y!2,s),P(x!2,f\sim 1) \leftrightarrow P(f\sim 0,y!1),P(x!1,f\sim 1,y!2,s),P(x!2,f\sim 1)@1,2$

'flip 101'  $P(f\sim 1,y!1),P(x!1,f\sim 0,y!2,s),P(x!2,f\sim 1) \leftrightarrow P(f\sim 1,y!1),P(x!1,f\sim 1,y!2,s),P(x!2,f\sim 1)@100,2$

### 4 P flips with CheY – note that all forwards are multiplied by 10 (one simple way to satisfy the ANC

### thermodynamic constraint, aka the Wegscheider condition)

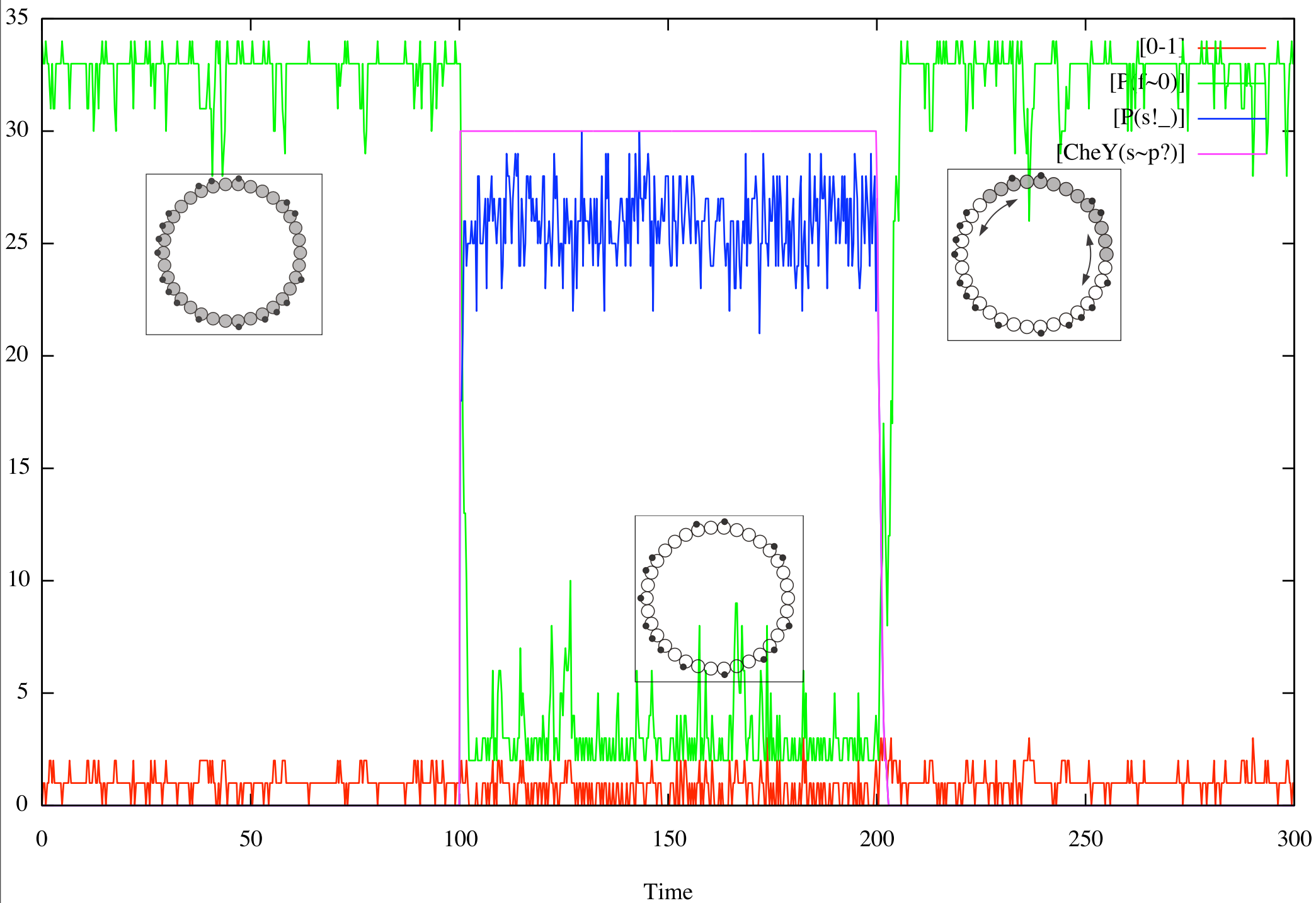
'flip 000b'  $P(f\sim 0,y!1),P(x!1,f\sim 0,y!2,s!_),P(x!2,f\sim 0) \leftrightarrow P(f\sim 0,y!1),P(x!1,f\sim 1,y!2,s!_),P(x!2,f\sim 0)@10,200$

'flip 100b'  $P(f\sim 1,y!1),P(x!1,f\sim 0,y!2,s!_),P(x!2,f\sim 0) \leftrightarrow P(f\sim 1,y!1),P(x!1,f\sim 1,y!2,s!_),P(x!2,f\sim 0)@10,2$

'flip 001b'  $P(f\sim 0,y!1),P(x!1,f\sim 0,y!2,s!_),P(x!2,f\sim 1) \leftrightarrow P(f\sim 0,y!1),P(x!1,f\sim 1,y!2,s!_),P(x!2,f\sim 1)@10,2$

'flip 101b'  $P(f\sim 1,y!1),P(x!1,f\sim 0,y!2,s!_),P(x!2,f\sim 1) \leftrightarrow P(f\sim 1,y!1),P(x!1,f\sim 1,y!2,s!_),P(x!2,f\sim 1)@1000,2$

10/1/2010 cs.ka sample=0.3000t.u



# conformation spread

The lower curve – tracking the Ising energy of the ring stays low at all time

– despite fraction of inactive P's ranging in  $[0, 1]$  depending on nb of CheY–Ps

NB: a Duke, Bray, Le Novere model; does not need a regular and/or permanent lattice



# home run!

- more physically realistic: seems to fit really well in this case (see Ref)
- less parameter-frenzy:
  - 10 reversible rules
  - 8 energy terms = 2 flips + (2+4) binds
  - 16 independent choices of kinetic rates “time scales”
- more structured approach: it really shines!
- esp. well suited for combinatorial molecular networks:  $10^{20}$ !